Lecture 1

1 Introduction to modelling

1.1 Areas of application

Mathematics is applied to a remarkably diverse range of topics, and is the natural language of science and technology. In physics, for example, is it used in *classical mechanics* (describing the motion of objects), *quantum mechanics* (describing the behaviour of atomic-scale objects) and *relativity* (the physics of the very large and the very fast). In chemistry, mathematical models describe how molecules interact in chemical reactions. In biology and medicine, mathematics is used to decode the human genome and in imaging our bodies. In economics and social sciences, mathematical models are widely used to predict stock prices or population changes.

1.2 What is a mathematical model?

A mathematical model *describes* a phenomenon using mathematics, and *predicts* how the phenomenon may take place in the future. The description may be *quantitative* (giving accuracate numbers) or *qualitative* (giving the overall behaviour and trends but not with numerical accuracy).

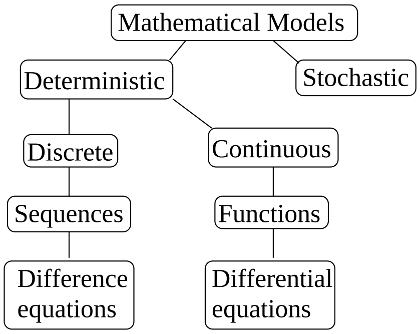
The use of predictive, quantitative mathematical models in guiding government policy during the 2001 Foot and Mouth epidemic is a well-known example.

A model is a set of equations, to be solved either by hand (analytically) or with a computer (numerically). The equations are written in terms of mathematical objects that correspond directly to physical quantities. If these objects change as part of the phenomenon they are generally called *variables*. If they are fixed they are generally called *parameters*. For a system that varies smoothly as time increases, for example, we use the variable t to describe time. If we describe a system that changes from year to year, say, we may use the variable n to number consecutive years. t is an example of a *continuous variable*; n is an example of a *discrete variable*.

1.3 Model classification

Models fall into two major classes.

- *Deterministic:* the future state of the system is completely determined (in principle) by previous behaviour.
- *Stochastic:* the system is subject to unpredictable, random fluctuations. These models involve probability and statistics.



In practice, the complexity of some deterministic models gives them effectively stochastic properties (e.g. the turbulent, unpredictable flow of a river is governed by deterministic equations). In this module we will consider only deterministic models.

These fall into two further categories.

- *Discrete models* use discrete variables, generally related by *difference equations*
- *Continuous models* use continuous variables, generally related by *differential equations*

1.3.1 A discrete model

As an example of a discrete model, consider a population of rabbits on an island. The variables are n, the number of years since 1980, and p_n, the rabbit population in year n ($n = 0, 1, 2, \ldots$). The set of numbers $p_0, p_1, p_2, \ldots$ is called a *sequence*. To model the growth in the population from year to year, we might try to use the *difference equation*

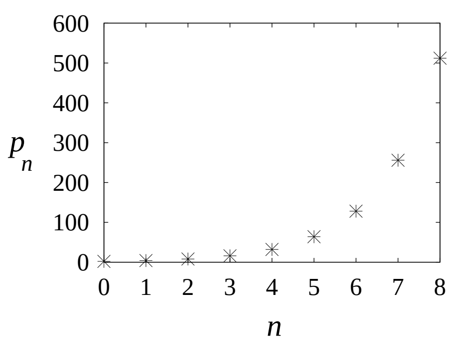
$$p_{n+1} = kp_n \qquad (n \geq 0),$$

for some constant $k > 0$. (k is an example of a *parameter*.)

Now

$$\begin{aligned} p_1 =& kp_0 \\ p_2 =& kp_1 = k^2 p_0 \\ p_3 =& kp_2 = k^3 p_0 \\ & \ldots \\ p_n =& k^n p_0; \end{aligned}$$

we have written down the solution for p_n *by inspection*. One can prove this is the correct solution using, for example, *induction* (see G11ACF).



The graph shows p_n when $p_0 = 2$ and $k = 2$.
Is this a realistic model? What features of a real rabbit population does it fail to describe?