1 Introduction to Modeling

1.1 Introduction to Dynamical Systems

In working through this text, you are going to learn to use mathematics to answer important questions about real situations, such as determining how much you can afford to borrow given your monthly salary or how to control your diet to achieve and maintain a desired weight. In other cases, you will use mathematics to get a better understanding of certain situations, such as how medicines build up and are eliminated from a body.

The situations studied in this text are those in which some quantity is changing over time. Such situations are **dynamic.** In Section 1.2, you will learn to model the situations using what is called a **discrete dynamical system.** A discrete dynamical system is an equation that relates a quantity at one point in time to the same quantity at an earlier point in time. For example, suppose an object is at a position of 10 feet from a starting point and that it is traveling away from the start at 2 feet per second. In one second it will be 12 feet from the start, in 2 seconds it will be 14 feet from the start, and so on.

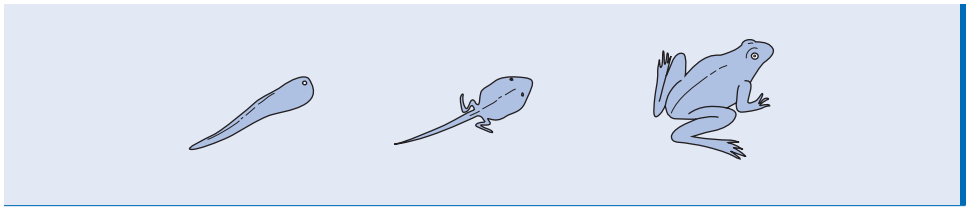
The situations studied in Section 1.3 involve one or more quantities that are changing over time. The change may involve the addition or subtraction of a constant amount in addition to the addition or subtraction of a constant proportion. One simple example would be a bank account to which a constant deposit is being added in addition to interest, which is a proportion of what was already in the account.

In Section 1.4, you will learn to generate tables and graphs using a calculator or computer to answer difficult but important questions about dynamic situations. The use of computers or calculators is overkill when studying simple situations, but for some of the more difficult situations in Section 1.4, and in the rest of the book, the use of computational technology will be indispensable.

Section 1.5 is devoted to the study of finance.

Dynamic refers to situations changing over time, such as, a frog going through its stages of life as in Figure 1.1.

FIGURE 1.1
A dynamic situation.



1.2 Examples of Modeling

In this section, you will learn the notation that will be used throughout the book and will use this notation to begin describing some simple dynamic situations, that is, situations in which something is changing. You will then study these situations using 2 different approaches. When studying the situations, I will point out some advantages and disadvantages of each approach.

Consider the following simple situation. Suppose you earn \$15 per hour tutoring mathematics. You wish to know how much money you will earn for tutoring different amounts of time. Table 1.1 gives some possible results.

TABLE 1.1 Amount earned for tutoring different numbers of hours.

hours worked	0	1	2	3	4
money earned	0	15	30	45	60

The first step in modeling is to assign variables. Let n represent the number of hours worked. The variable n is called the **independent variable** and usually represents time.

Let u represent the amount of money earned tutoring. The variable u depends on n, the number of hours worked, and will usually be written as $u(n)$, meaning the amount of money earned for tutoring n hours. Therefore u is called the **dependent variable.** The dependent variable is a **function** of the independent variable. Table 1.2 gives the value of the dependent variable for several values of the independent variable. For example, $u(3) = 45$ is read as "the amount of money earned for working 3 hours is 45 dollars."

TABLE 1.2 Values of $u(n)$ for several values of n.

hours worked	$n = 0$	$n = 1$	$n = 2$	$n = 3$	$n = 4$
money earned	$u(0) = 0$	$u(1) = 15$	$u(2) = 30$	$u(3) = 45$	$u(4) = 60$

The dependent variable represents the size or amount of what is changing as the independent variable changes, such as the distance our car is from home after several hours, the amount of money in an account after several years, or the prevalence of a particular genetic trait after several generations.

When modeling, you should always write a specific description of what each variable represents. In the case of tutoring, you might give the following descriptions of the variables.

Independent variable: n represents the integer number of hours worked.

Dependent variable: $u(n)$ represents the amount of money earned working n hours.

You should get into the habit of writing precise descriptions of what the independent and dependent variables represent. Do not make vague descriptions, such as n is time or $u(n)$ is money, because these descriptions are incomplete and will lead to problems later. Also note that the description of the dependent variable should refer back to the independent variable.

Different tutors could have different hourly rates. For example, 1 tutor might charge \$10 per hour and another tutor \$20 per hour. We could let r represent the hourly rate for a tutor. Since you charge \$15 per hour, $r = 15$ when modeling your earnings. The variable r is called a **parameter** that varies depending on the particular situation.

The **domain** of the function is the set of n-values for which $u(n)$ is defined. In this book, the domain will consist of the set of n-values for which the context makes sense, usually the positive integers $n = 1, 2, \ldots$ or the nonnegative integers $n = 0, 1, \ldots$. In the tutoring example, the domain is $n = 0, 1, \ldots$.

Plotting the points $(0, u(0))$, $(1, u(1))$, $\ldots$ on a graph helps in understanding the relationship between the independent and dependent variables. The n-values are in the horizontal direction and the u-values in the vertical direction. For the tutoring example, the points $(0,0)$, $(1,15)$, $(2,30)$, $\ldots$, $(20,300)$ are plotted on Figure 1.2. Connecting consecutive points with a line helps us see a pattern. But remember that the only points that have meaning are those for which the independent variable is an integer. Such a graph is called a **time graph** because the dependent variable is plotted over time.

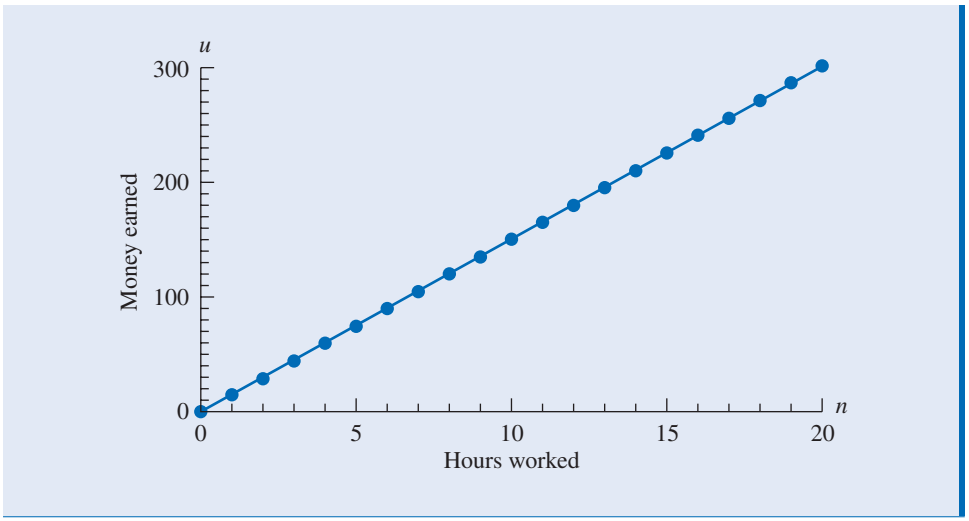


FIGURE 1.2
Graph of hours worked versus money earned.